TEACHING PHILOSOPHY

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The great utility and beauty of mathematics is its universality. Mathematics is generally expressed as a formal language, but humans beings do not learn this language until much later in life. The basic concepts of space, change, and number are familiar to us in infancy. We learn to count very soon after we learn how to distinguish objects and ourselves from others. We learn about space by moving around in it and exploring the world. We learn about change as we grow over time. It is nearly impossible to simply *avoid* learning about mathematics, because it is interwoven with what it means to be a human being. At some point, however, it becomes difficult for folks to see the math underlying and embedded in the world around them. Instead, they settle for only the bare minimum of practical math required to get by. The world becomes a much richer place when these connections are evident - the fluid mechanics equations modeling the swirls in your coffee, the structure of the weighted network of road connections in your hometown, and the frequencies emanating from a violin described using representation theory. As an educator, I seek to instill in my students the sheer awe and wonder at the relevance of mathematics, rather than dismiss it as irrelevant abstract nonsense.

It is often said that the power in mathematics comes from its abstraction. That is partly true. Indeed, abstraction allows us to forget certain aspects of the structure of real world objects while retaining what's important in order to see similarities that might otherwise be invisible. For example, a bicycle wheel and car wheel are different but have many things in common, and we might first note they're both wheels, they both can have a fixed angular momentum. However, a mathematician might go further, and abstract them all the way to a circle. From this perspective, we see that there is in fact an infinite group of symmetries they have in common. Further, this group has the structure of a manifold, and we find examples of this group all over the world.

While abstraction is a valuable tool, mathematics really shines when understand structures well enough to perform computations, and communicate them in a clear way. We need to *represent* objects in some way, or model them in such a way that we can write them down. For example, we can view groups as an abstract way to encode the symmetries of an object. Starting with a simple object like a square, we might note that we can rotate it ninety degrees, and each time we do this nothing noticeable changes. If we label the corners, we can now tell, but performing the rotation four times again changes nothing.

Date: January 11, 2026.

We could also flip the square over a diagonal. We might note there are eight total *symmetries*. However, these transformations take place in the plane, and we can ask what these transformations do to vectors in the plane. This gives us a two dimensional representation consisting of eight 2×2 matrices. We can thus put these into the computer, compute, visualize, and communicate almost all there is to know about this group.

In short, mathematics shines when we translate the abstract to the concrete. Once we see that the symmetries of something is a circle, we might consider *functions* on the circle, introducing the notion of periodicity. We can ask how many times a function wraps around a circle, giving us an integer value called the winding number. In fact, each of these numbers parameterizes a one dimensional representation of the circle, and these special functions are called *characters*. Together these characters allow one to decompose any periodic function into a weighted sum of characters, which we can interpret as frequencies. This is one of the most powerful techniques in all of mathematics called the *Fourier transform*. It is the way we separate out the sounds of a piano into its tones and recognize its timbre.

While teaching, I like to give as many concrete examples as possible. While the theoretical background is important, actually understanding a complicated object or equation in mathematics requires getting your hands dirty. Students need to spend time with them, both guided by teachers, with peers, and by themselves to fully absorb a topic. Simply reading about it, or writing down notes is just not enough. For that reason, I like to keep homework very regular and as light as possible. I would much rather someone solve problems related to the lecture material very soon after they see it when it's fresh, rather than do a batch of problems right before the exam.

I believe in keeping expectations high but meeting students where they are. There will, inevitably, be students that simply do not want to or are unable to pass the course, and I'd like to keep those numbers as low as feasibly possible. I think it's important to be available for office hours outside of the class to help students figure things out. I have seen many students that just need to be told they're moving in the right direction, and have someone confirm that they're doing the right thing. I like to keep my exams "tough but fair", and would rather have a challenging exam that is appropriately curved than an exam that's too easy that everyone aces.

I treat students as adult learners who are in a college class for a reason - to learn the material. I do not have much patience for folks distracting themselves or being distracting in class. I expect students to arrive on time, and I will give my heart and soul to ensure I'm doing my absolute best to teach the material, and I hope that they are doing their best to learn and absorb the material. Students will inevitably make mistakes, and as much as possible I want to enable students to learn from their mistakes within the parameters of the syllabus.

Above all, I want students to not fear mathematics, but embrace it as a friend and a tool to understand the world around them. By the end of a course, they should feel that a subject has been demystified, and the fog of uncertainty should have cleared. In ideal scenarios, they should feel that they are now capable of teaching it to others.

n.b. With respect to artificial intelligence, this is now a permanent feature of our world. These tools are very powerful, and essentially represent (in my view) a compression of much of the knowledge in the world that we have bothered to write down. It is possible to use the tools to solve a calculus or differential equations problem, surely. But that does not mean that a student will understand the problem being solved, and this has severe consequences for their future abilities to solve more complicated problems, or even prompt the machine with a more complicated problem. Thus, I feel that if our goal is to learn a subject, no student should be uploading questions directly to the machine to solve. Using it to talk about the material is okay, using it to generate examples to learn from is okay, but students will not actually learn differential equations without solving differential equations. There is no substitute for this. Therefore it now seems necessary to do completely closed book and closed laptop exams. If AI is used to do homework, then it very likely that student will fail the exam.